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Static potential in scalar QED₃ with non-minimal coupling

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Abstract

Here we compute the static potential in scalar QED₃ at leading order in $1/N_f$. We show that the addition of a non-minimal coupling of Pauli-type ($\epsilon_{\mu\nu\alpha} j^\mu \partial^\nu A^\alpha$), although it breaks parity, it does not change the analytic structure of the photon propagator and consequently the static potential remains logarithmic (confining) at large distances. The non-minimal coupling modifies the potential, however, at small charge separations giving rise to a repulsive force of short range between opposite sign charges, which is relevant for the existence of bound states. This effect is in agreement with a previous calculation based on Möller scattering, but differently from such calculation we show here that the repulsion appears independently of the presence of a tree level Chern–Simons term which rather affects the large distance behaviour of the potential turning it into a constant.

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1. Introduction

An important problem in high energy physics is the lack of a rigorous proof of colour confinement in 4D QCD. Different techniques have been used to tackle this problem. We can mention lattice simulations [1], supersymmetry [2] and lower dimensional models [3–5].

In order to investigate the contribution of the matter fields to this problem we integrate over such fields in the path integral and derive an effective action for the vector bosons; solving the equations of motion of this quantum action we can compute the potential between two static charges separated by a distance L . A monotonically increasing potential as $L \rightarrow \infty$ signalizes confinement. This route has been followed in [6–9] in the case of QED₃. In that model, if we work with two-component fermions, the fermion mass term breaks parity and a Chern–Simons term is dynamically generated leading to an important change in the analytic

structure of the photon propagator which turns the classically confining logarithmic potential into a constant at large distances.

In the case of scalar QED₃ we have a different scenario since its mass term, like the rest of the Lagrangian, is parity symmetric and no parity breaking term is dynamically generated, so the classical logarithmic potential survives at quantum level. Therefore, it is expected that the inclusion of parity breaking terms in the Lagrangian would strongly modify the static potential. A natural possibility to be considered is a non-minimal coupling of Pauli type which breaks parity but preserves gauge invariance. This term is rather simple in $D = 3$, where the dual field strength ($F_\mu = \epsilon_{\mu\nu\alpha} A^\nu \partial^\alpha A^\alpha$) is a pseudo-vector and the addition of the non-minimal coupling amounts to the replacement $eA_\mu \rightarrow eA_\mu + \gamma F_\mu$, where γ is the non-minimal coupling constant which has negative mass dimension. This term has been considered before in the literature of QED₃ and scalar QED₃, see e.g. [9–16].

Another motivation for the inclusion of the non-minimal coupling comes from [12–14], where there are indications, see however [15], that the coupling of a gauge field to fermions via a Pauli term could give rise to anyons with no need of a Chern–Simons term. Since the change of statistics is a long-range phenomenon and the Chern–Simons term indeed changes the static potential at large distances, we would like to include the Pauli-type interaction in order to check, at least in some approximation, if it could really produce large distance effects.

A further point concerns previous calculations in the literature. It has been claimed in [16, 17] that the effect of the non-minimal coupling on the static potential only appears if a Chern–Simons term is present. This is apparently not the case of QED₃ with four-component fermions where no Chern–Simons term is generated but still there is some influence of the Pauli-type term on the static potential at low distances [9]. It is important to remark, however, that here and in [9] one works at leading order in $1/N_f$ which requires the calculation of the one-loop vacuum polarization diagram, while the calculations of [16, 17] are based on the one-photon exchange diagram at tree level (Möller scattering) in the non-relativistic limit. In order to control the effect of the Chern–Simons term and compare our results to [16, 17] we introduce here, besides the Pauli-type term, a Chern–Simons term at tree level with an arbitrary coefficient.

We have already mentioned that the Pauli term demands a coupling constant with negative mass dimension (non-renormalizable), so we found suitable to use $1/N_f$ expansion since there are some arguments [18] in favour of the $1/N_f$ renormalizability of such interaction. In the following section we start by integrating over the N_f scalar fields at leading order in $1/N_f$. Then, we analyse the analyticity properties of the corresponding photon propagator. In section 3 we minimize the effective action and compute the static potential $V(L)$ numerically for a finite scalar mass and analytically in the limit $m \rightarrow \infty$. We draw some conclusions in section 4.

2. The photon propagator at $N_f \rightarrow \infty$

Our starting point is to integrate over the N_f scalar fields ϕ_r , $r = 1, 2, \dots, N_f$ in the partition function below:

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu \exp\left(i \int d^3x \mathcal{L}(A^\mu, j_{\text{ext}}^v)\right) \prod_{r=1}^{N_f} \mathcal{D}\phi_r^* \mathcal{D}\phi_r \exp\left(-\frac{i}{2} \int d^3x \phi_r^* [D^\mu D_\mu + m^2] \phi_r\right) \\ &= C \int \mathcal{D}A_\mu \exp\left(i \int d^3x \mathcal{L}(A^\mu, j_{\text{ext}}^v) - N_f \text{Tr} \ln[D^\mu D_\mu + m^2]\right), \end{aligned} \quad (1)$$

where C is a numerical constant and

$$\mathcal{L}(A^\mu, j_{\text{ext}}^\nu) = -\frac{1}{4}F_{\mu\nu}^2 - \frac{\theta}{2}\epsilon_{\mu\nu\alpha}A^\mu\partial^\nu A^\alpha + \frac{\zeta}{2}(\partial_\mu A^\mu)^2 - A_\nu j_{\text{ext}}^\nu. \quad (2)$$

The external current corresponding to a static charge Q at the point $(x_1, x_2) = (L/2, 0)$ is given by $j_{\text{ext}}^\nu = Q\delta(x_2)\delta(x_1 - \frac{L}{2})\delta^{\nu 0}$. Later on, the interaction energy of a couple of charges $-Q$ and Q separated by a distance L will be calculated via $V(L) = -QA_0(x_1 = -L/2, x_2 = 0)$, where $A_0(x_\nu)$ will be obtained minimizing the effective action coming from (1). The covariant derivative $D_\mu\phi = \partial_\mu\phi - ie\phi A_\mu/\sqrt{N_f} - i\gamma\phi F_\mu/\sqrt{N_f}$ includes the non-minimal coupling constant γ which has negative mass dimension $[\gamma] = -1/2$ while $[e] = 1/2$ and $[\theta] = 1$. The dual of the strength tensor is defined here as $F_\mu = \epsilon_{\mu\nu\alpha}\partial^\nu A^\alpha$.

The next step is to evaluate the trace of the logarithm perturbatively in $1/N_f$. We have two types of interaction vertices coming from $\mathcal{L}_{\text{int}}^{(1)} = i(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)(eA^\mu + \gamma F^\mu)/\sqrt{N_f}$ and $\mathcal{L}_{\text{int}}^{(2)} = i\phi^*\phi(eA^\mu + \gamma F^\mu)^2/N_f$. Thus, the leading contribution in $1/N_f$ would come from just one vertex of the first type; however, since it involves derivatives of the scalar fields the Feynman rules for scalar QED include a factor $p_\mu^{\text{in}} + p_\mu^{\text{out}}$ where those are the incoming and outgoing momenta of the scalar fields. Therefore the diagram (tadpole) will be proportional to the integral $\int d^3p(p_\mu^{\text{in}} + p_\mu^{\text{out}})/(p^2 - m^2) = \int d^3p 2p_\mu/(p^2 - m^2)$ which vanishes in the dimensional regularization adopted here. The next leading contribution includes either two vertices of the first type or one vertex of the second type. Both contributions will be independent of N_f due to the overall factor N_f in front of the logarithm in (1) and will survive the limit $N_f \rightarrow \infty$. The next contribution would come from three vertices of the first type and would be of order $1/\sqrt{N_f}$ so it vanishes if $N_f \rightarrow \infty$. Such contributions and higher ones will be neglected henceforth. In conclusion we have, up to an overall constant, $Z = \int \mathcal{D}A_\mu e^{iS_{\text{eff}}}$ where

$$S_{\text{eff}} = \int d^3x \mathcal{L}(A^\mu, j_{\text{ext}}^\nu) - \frac{i}{2} \int d^3k (e\tilde{A}^\mu(k) + \gamma\tilde{F}^\mu(k))T_{\mu\nu}(e\tilde{A}^\nu(-k) + \gamma\tilde{F}^\nu(-k)). \quad (3)$$

The quantities $\tilde{A}_\mu, \tilde{F}_\nu$ are Fourier transforms and

$$T_{\alpha\beta} = -2g_{\alpha\beta}I^{(1)} + I_{\alpha\beta}^{(2)}. \quad (4)$$

Using dimensional regularization we have obtained for the Feynman integrals

$$I^{(1)} = \int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2 - m^2} = i\frac{m}{4\pi} \quad (5)$$

$$I_{\alpha\beta}^{(2)} = \int \frac{d^3p}{(2\pi)^3} \frac{(2p+k)_\alpha(2p+k)_\beta}{(p^2 - m^2)[(p+k)^2 - m^2]} = \frac{im}{8\pi}[4g_{\alpha\beta} - 2zf_2\theta_{\alpha\beta}], \quad (6)$$

with $z = k^2/4m^2$ and $\theta_{\alpha\beta} = g_{\alpha\beta} - k_\alpha k_\beta/k^2$. In the region $0 \leq z \leq 1$ we have

$$f_2 = \frac{1}{z} \left[1 + \frac{1-z}{2}f_1 \right] = \frac{2}{3} + \frac{2}{15}z + \frac{2}{35}z^2 + \dots \quad (7)$$

$$f_1 = -\frac{1}{\sqrt{z}} \ln \frac{1+\sqrt{z}}{1-\sqrt{z}}. \quad (8)$$

Above the pair creation threshold ($z > 1$) the integral $I_{\alpha\beta}^{(2)}$ develops a real part which will be neglected henceforth. For future use we have given the large mass expansion for f_2 . The static potential requires the expression for the effective action for $z < 0$ which can be obtained

by analytically continuing (7) and (8). Namely, with $\tilde{z} = -z > 0$ we have

$$\tilde{f}_2 = -\frac{1}{\tilde{z}} \left[1 + \frac{1+\tilde{z}}{2} \tilde{f}_1 \right] = \frac{2}{3} - \frac{2}{15} \tilde{z} + \frac{2}{35} \tilde{z}^2 + \dots \quad (9)$$

$$\tilde{f}_1 = -\frac{2}{\sqrt{\tilde{z}}} \arctan \sqrt{\tilde{z}}. \quad (10)$$

Our result for $T_{\alpha\beta}$ is in agreement with [19] and it is transverse $k^\alpha T_{\alpha\beta} = 0 = T_{\alpha\beta} k^\beta$ in accordance with gauge invariance. Now we can write down the effective action for scalar QED₃ including vacuum polarization effects:

$$S_{\text{eff}} = \int d^3x \left\{ -\frac{1}{4} F^{\mu\nu} \left[1 - \frac{\gamma^2 \square f_2}{16\pi m} + \frac{e^2 f_2}{16m\pi} \right] F_{\mu\nu} + \frac{\zeta}{2} (\partial_\mu A^\mu)^2 - \frac{\theta}{2} \epsilon_{\mu\nu\alpha} A^\mu \partial^\nu A^\alpha - \frac{e\gamma}{16m\pi} \epsilon_{\mu\nu\alpha} A^\mu \partial^\nu \square f_2 A^\alpha - A_\nu j_{\text{ext}}^\nu \right\}, \quad (11)$$

where $f_2 = f_2(-\square/4m^2)$ is given in (7) and (9). Note that, besides the tree level Chern-Simons term, another parity breaking term appears in (11) due to the magnetic moment interaction. Although the action (11) is non-local it can be made local in the large mass limit $m \rightarrow \infty$ as in [20]. Introducing the dimensionless constants

$$c_1 = \frac{e^2}{16\pi m}, \quad c_2 = \frac{e\gamma}{8\pi}, \quad c_3 = \frac{\theta}{2m}, \quad (12)$$

taking $m \rightarrow \infty$ while keeping the dimensionless constants finite, the only effect of the vacuum polarization is a finite renormalization of the Maxwell term, i.e.,

$$S_{\text{eff}}(m \rightarrow \infty) = \int d^3x \left[-\frac{1+2c_1/3}{4} F_{\mu\nu}^2 - \frac{\theta}{2} \epsilon_{\mu\nu\alpha} A^\mu \partial^\nu A^\alpha + \frac{\zeta}{2} (\partial_\mu A^\mu)^2 - A_\nu j_{\text{ext}}^\nu \right]. \quad (13)$$

On the other hand, for finite mass we can write

$$S_{\text{eff}} = \int d^3x d^3y \left[A^\mu(x) \frac{D_{\mu\nu}^{-1}(x,y)}{2} A^\nu(y) - A_\nu j_{\text{ext}}^\nu \delta^{(3)}(x-y) \right] \quad (14)$$

$$= \int \frac{d^3k}{(2\pi)^3} \tilde{A}^\mu(k) \frac{\tilde{D}_{\mu\nu}(k)^{-1}}{2} \tilde{A}^\nu(-k) - \int d^3x A_\nu j_{\text{ext}}^\nu, \quad (15)$$

where the photon propagator in momentum space is given by

$$\tilde{D}_{\mu\nu} = a(g_{\mu\nu} - \theta_{\mu\nu}) + b\theta_{\mu\nu} + c\epsilon_{\mu\nu\alpha} k^\alpha, \quad (16)$$

with

$$a = \frac{1}{\xi k^2} \quad (17)$$

$$b = -\frac{c_1(D_+ + D_-)}{8m^2 \sqrt{z} D_+ D_-} \quad (18)$$

$$c = -i \frac{c_1(D_+ - D_-)}{16m^3 z D_+ D_-} \quad (19)$$

$$D_\pm = \sqrt{z} [c_1 + (c_1 \pm \sqrt{z} c_2)^2 f_2] \pm c_1 c_3 \equiv g_\pm(z) \pm c_1 c_3. \quad (20)$$

Now we are able to analyse the analyticity properties of the photon propagator. First of all, we note that the massless pole $z = 0$ in the denominator of (19), which is typical of

a Maxwell–Chern–Simons theory, is a gauge artefact. It disappears from gauge invariant correlators involving the field strength F_μ . It can be shown [21] to have a vanishing residue (non-propagating mode). Since the factor \sqrt{z} in the denominator of (18) is cancelled out by the numerator, the only possibilities for poles in the propagator stem either from $D_+ = 0$ or $D_- = 0$. Due to $c_1 > 0$ and $2/3 \leq f_2 < 1$, we have $g_\pm(z) > 0$ and consequently we can only have $D_+ = 0$ or $D_- = 0$ for $c_3 < 0$ or $c_3 > 0$, respectively. We never have two poles at the same time. In the absence of the Chern–Simons term, i.e., $c_3 = 0$, the product D_+D_- will be proportional to z and we are left with one massless pole $z = 0$. Since $\lim_{z \rightarrow 0} z(a + b) = -1/(1 + 2c_1/3) < 0$ the residue at this pole will be positive and this represents a physical massless photon which will be responsible for a long-range logarithmic static potential. On the other hand, if $c_3 \neq 0$, since the denominator D_+D_- is symmetric under $c_2 \rightarrow -c_2, c_3 \rightarrow -c_3$, it is enough to consider only $D_+ = 0$ assuming $c_3 < 0$; the other case $D_- = 0$ with $c_3 > 0$ follows from the symmetry. Numerically, we have checked that whatever sign we choose for c_2 the function $g_+(z)$ is monotonically increasing and satisfies $g_+(z) > 0$; consequently its maximum is $g_+(1)$. Therefore, see (20), if $c_3 < -g_+(1)/c_1 = [(c_1 + c_2)^2 + c_1]/c_1$, then we have no poles and so no particle in the spectrum. On the opposite, if $-g_+(1)/c_1 < c_3 < 0$ we are always able to find numerically one massive pole for some $0 < z < 1$ such that $D_+ = 0$ which is a typical effect of a Chern–Simons term [22]. As we move towards the left limit value $c_3 \rightarrow -g_+(1)/c_1$, the photon mass increases to the point where it reaches the pair creation threshold $k^2 = 4m^2$ at $c_3 = -g_+(1)/c_1$. Due to the symmetry $c_2 \rightarrow -c_2, c_3 \rightarrow -c_3$ we conclude that whenever the tree level Chern–Simons term is present and its coefficient is not too much negative or too much positive ($|c_3| < g_+(1)/c_1$) we have one massive physical (positive residue) photon and if the Chern–Simons term is absent we have one physical massless photon.

It is remarkable to find a no poles region in the photon propagator. One might think that this is due to some convergence problem of the $1/N_f$ expansion which has been introduced because of the non-renormalizable non-minimal coupling c_2 . However, even if $c_2 = 0$ the Chern–Simons coefficient must obey an upper bound $|c_3| < g_+(1)/c_1 = 1 + c_1$ in order to have a physical pole in the photon propagator at one-loop level. By analytically continuing, see (9), the expression for the propagator to the region $z = k^2/4m^2 < 0$, we have checked that tachyons can only appear for a special fine tuning of the coupling constants for which we did not find any special interpretation, namely, the tachyonic pole must be a solution of $\tilde{z}\tilde{f}_2 = c_3/(2c_2)$ and this solution must be such that $c_1^2c_3 = -\tilde{z}c_2(2c_1 - c_2c_3)$; although explicit numerical solutions are possible we have found those fine tuned cases rather artificial. In particular, they have apparently no relationship with the no-pole region ($|c_3| > g_+(1)/c_1$) and will be disregarded in this work. In the following section we use the photon propagator as an input to calculate the static potential $V(L)$.

3. The static potential $V(L)$

Minimizing the effective action (14) we obtain

$$A_\beta(y) = \int d^3x D_{\beta\alpha}(y, x) j_{\text{ext}}^\alpha(x) \quad (21)$$

$$= \int \frac{d^3k}{(2\pi)^3} \tilde{D}_{\beta\alpha}(k) \int d^3x e^{ik \cdot (y-x)} j_{\text{ext}}^\alpha(x). \quad (22)$$

Since the external current is time independent, in (22) there will be a factor $\int dx_0 e^{-ik_0x_0} = 2\pi\delta(k_0)$ which allows an exact integration over k_0 , implying $k^\mu k_\mu = -k_1^2 - k_2^2 = -\vec{k}^2 < 0$;

consequently $z < 0$ and we need the analytic continued functions \tilde{f}_2 instead of f_2 . The integrals over x_1 and x_2 can also be readily done using the delta functions in the external current. The angle part of the integral $dk_1 dk_2 = k dk d\theta$ gives rise to the Bessel function $J_0(kL)$. Thus, we are left with the radial integral over $k = \sqrt{k_1^2 + k_2^2}$. Placing the negative charge $-Q$ at $(y_1, y_2) = (-L/2, 0)$ we have

$$\begin{aligned} V(L) &= -QA_0(y_1 = -L/2, y_2 = 0) \\ &= -\frac{Q^2}{2\pi} \int_0^\infty dk k \tilde{b} J_0(kL). \end{aligned} \quad (23)$$

The tilde in the expression \tilde{b} stands for the analytic continuation of (18) to $z < 0$.

Now we discuss some special cases starting with the pure scalar QED₃ where $c_3 = 0 = c_2$. In this case $k\tilde{b}J_0(kL) = J_0(kL)/[k(1 + c_1\tilde{f}_2)]$ since $\tilde{f}_2(k = 0) = 2/3$ and $J_0(0) = 1$ we have an infrared divergence at $k = 0$ and the integral (23) is divergent as it stands. We make a subtraction in order to get rid of this infrared divergence and define

$$V(L) - V(L_0) = -\frac{Q^2}{2\pi} \lim_{x \rightarrow 0} \int_x^\infty dk k \tilde{b} [J_0(kL) - J_0(kL_0)]. \quad (24)$$

In general the integral (24) must be calculated numerically; one exception is the large mass limit $m \rightarrow \infty$. In this case $k\tilde{b} \rightarrow 1/[k(1 + 2c_1/3)]$ and the integral can be calculated exactly [23]

$$[V(L) - V(L_0)]_{m \rightarrow \infty} = \frac{Q_R^2}{2\pi} \ln \left(\frac{L}{L_0} \right), \quad (25)$$

where

$$Q_R = \frac{Q}{\left[1 + \frac{e^2}{24\pi m}\right]^{1/2}}. \quad (26)$$

The classical potential is given by (25) with Q_R replaced by the bare charge Q . Therefore, the sole effect of the vacuum polarization is a finite renormalization of the charge. The situation is similar to QED₃ with four-component fermions where no Chern–Simons term is dynamically generated, the only difference is the renormalized charge which is $Q/[1 + e^2/(6\pi m)]$ instead of (26). Thus, the renormalization factor is larger for fermions than for scalars. For finite mass the potential must be calculated numerically. We plot¹ the results in figure 1 for the masses $m = 1$ and $m = 0.01$ and compare with the classical result and the result of [9] for four-component fermions. We note that the finite renormalization due to the vacuum polarization is always stronger for fermions than for scalars, and its effect increases with the mass of the matter fields. For both scalar QED₃ and QED₃ we see in figure 1 that the numerically calculated static potential at $m = 1$ is already very close to the analytic result (solid lines) obtained in the limit $m \rightarrow \infty$.

Next, we check the effect of the non-minimal coupling $c_2 \neq 0$ in the absence of the Chern–Simons term ($c_3 = 0$). In figure 2 we see that for $L \rightarrow \infty$ the effect of the non-minimal coupling in the vacuum polarization disappears and the potential becomes logarithmic which can be explained technically by the fact that the Bessel function $J_0(kL)$ oscillates with decreasing amplitude as $L \rightarrow \infty$ and so the integral will be dominated by the pole at the origin $k = 0$ which makes the higher derivative, see (11), contribution of the non-minimal coupling negligible. However, in a finite range close to $L = 0$ the non-minimal coupling gives rise to a surprising repulsive force in a much similar way to what happens in the case of four-component fermions in [9]. Such repulsive force may play an important role in the

¹ In all figures in this work the symbol V stands actually for the difference $V(L) - V(L_0)$.

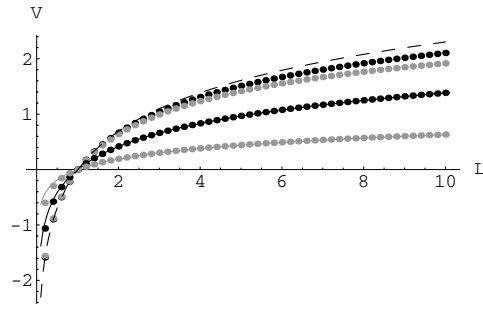


Figure 1. The static potential for pure scalar QED₃ (dark dots) and pure QED₃ with four-component fermions (light dots). The dashed line corresponds to the classical potential. We have fixed $(c_1, c_2, c_3) = (1, 0, 0)$ and $m = 0.01$ for the two dotted curves closer to the classical potential while we have $m = 1$ for the farther curves which overlap with the $m \rightarrow \infty$ analytic result (solid curves).

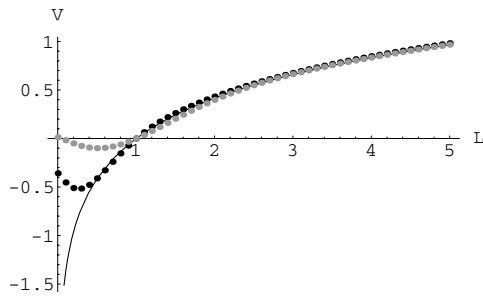


Figure 2. The static potential for scalar QED₃ without the tree level Chern–Simons term ($c_3 = 0$). The solid line corresponds to pure scalar QED₃ ($c_2 = 0$) while the dark (light) dots to $c_2 = 2$ ($c_2 = 4$). We have assumed $m = 3$ and $c_1 = 1$.

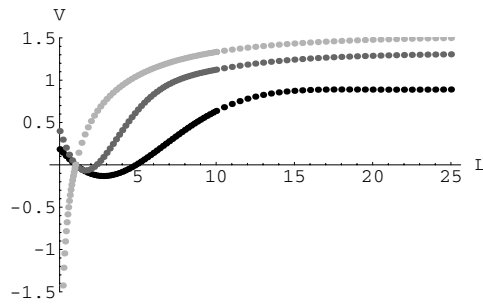


Figure 3. The static potential for scalar QED₃ with the tree level Chern–Simons term ($c_3 = 1$). From the lightest to the darkest curve we have $c_2 = 0, 0.4, 0.8$. We have assumed $m = 1 = c_1$.

existence of bound states. Differently from the calculation based on the Möller scattering [11] we see here effects of the non-minimal coupling even in the absence of the Chern–Simons term.

Now we turn on the Chern–Simons term ($c_3 \neq 0$). As we see in figure 3, the potential $V(L) - V(L_0)$ tends to the constant $-V(L_0)$ as $L \rightarrow \infty$ like the case of QED₃ with two-component fermions, see [6], where a Chern–Simons term is dynamically generated.

Once again, a repulsive force appears for small separations as we switch on the non-minimal coupling. As $L \rightarrow \infty$ the only effect of the non-minimal coupling is to change the constant $-V(L_0)$. Although, the plot in figure 3 has been made for $m = 1$ and $c_1 = 1$ we have checked that the same form of the potential persists for other values of those constants. In summary, the effect of the non-minimal coupling is qualitatively the same in the presence of a Chern–Simons term.

4. Discussions and conclusion

In the case of QED₃ with two-component fermions, it is well known that a Chern–Simons term is dynamically generated which makes the photon massive and turns the classical confining logarithmic potential into constant at large distances. In scalar QED₃, the mass term for the scalars is parity invariant and no Chern–Simons term is dynamically generated and so the classical logarithmic potential survives vacuum polarization effects. Here we have explicitly confirmed that fact and analysed the effect of adding a parity breaking non-minimal coupling term of Pauli type as well as a tree level Chern–Simons term. It turns out that the non-minimal coupling by itself neither affects the analytic properties of the photon propagator nor changes the large distance behaviour of the static potential which is unexpected from the point of view of the interpretation that this term may originate anyons with no need of a Chern–Simons term, see [12–14] but see also [15]. On the other hand, at small charge separations the non-minimal coupling gives rise to a repulsive force between opposite sign charges which has been observed before in [16] by computing the one-photon exchange diagram (Möller scattering) and taking the non-relativistic limit. Notwithstanding, the effect found in [16] only appears in the presence of the tree level Chern–Simons term and its attractive or repulsive nature depends on the sign of $(1 - \gamma\theta/e)$ in contrast to what we have found here where the non-minimal coupling influence is present, see figure 2, even if $\theta = 0$ and its effect is always repulsive independently of the sign of γ or θ .

Concerning the tree level Chern–Simons term, as expected, it gives mass to the photon and shifts the zero momentum pole in the integral involved in the static potential (24). The absence of a singularity in the integration path allows us to take the limit $L \rightarrow \infty$ before performing the integral and so it will vanish as a consequence of $J_0(x \rightarrow \infty) \rightarrow 0$. This effect of the Chern–Simons term was certainly not surprising. However, it is remarkable that we found an upper bound for the absolute value of the Chern–Simons coefficient in order to have a physical pole in the photon propagator at one-loop level. As we increase such absolute value the photon mass increases and penetrates the real pair creation region $k^2 \geq 4m^2$ for finite values of the coupling constants of the theory. We can mention that this situation is not peculiar to scalar fields since we have noted in [8] that it happens also in QED₃ with two-component fermions. In that case, if $c_1 = e^2/(16\pi m) \geq 1$, there will be no poles in the photon propagator at one-loop level. However, one could argue that c_1 is a dimensionless constant which controls the perturbative expansion (for $N_f = 1$) such that the upper bound could be understood as a limit for the perturbation theory. This argument does not work for scalar QED₃ even if we drop the non-minimal coupling ($c_2 = 0$) since the upper bound increases with c_1 which makes the latter case more intriguing.

At last, we note that the static potential in pure scalar QED₃ without tree level Chern–Simons term has been studied in [24] where the authors conclude that the potential is of screening type and even fractional charges can be fully screened. However, the authors of [24] have neglected terms of order e^2/m which have been considered here. Besides, they have gone above the pair creation threshold and made use of variational methods altogether with

a peculiar ansatz for the two-particle wavefunction, so we can hardly compare their findings with our results obtained below the pair creation threshold.

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